Set Membership

Before we start on "set membership" let us recall five things.

1. A set is just a collection of different "things" where "things" can be almost anything.
2. We often name sets with capital letters, such as A, B, C, and so on. We note that we will quickly run out of such letters so we need to be prepared to reuse the letters for different problems.
3. One way to define a set is to list all of the elements of a set. We do this by listing the elements, not necessarily in any special order, between braces and separated by commas. Thus, we might have the set A={1,3,5,7,9}, or the set B={dog,cat,bird}, or the set D={4, 8, 12, 16,…,44, 48}, or an infinite set F={…,343, 49, 7, 1, 1/7, 1/49, 1/343,…}.
4. An alternative method for defining a set is to describe a rule that tells us if something is in the set or not in the set. The formal version of this is called "set-builder notation". This notation again uses braces for the outside of the definition, but it starts with a variable, perhaps x, which is then followed by a vertical bar, |, which is read as "such that", and that is followed by the rule that we need to apply. Thus, we might have set G={x|x is an odd whole number between 0 and 10}, or H={x|x is one of the three most popular household pets}, or J={y|y is a multiple of 4 between 3 and 50}, or K={z|z is equal to 7 raised to an integer power}. Note that A and G are the same set, B and H are the same set, D and J are the same set, and F and K are the same set.

5. Since a set is a collection of different things, it makes no sense to have something in a set twice. Something is either in a set or it is not.

Given all of that, we can now talk about set membership. We want a short way to write something like "7 is an element of the set A" or "11 is not an element of set A". We have a symbol, namely $\in$, to represent "is an element of", and another symbol, namely $\notin$, to represent "is not an element of". Thus, we could say $7 \in A$, or we could say $11 \notin A$. Here are some more examples: hamster$\notin$H, $7 \notin D$, $16 \in D$, $32 \in D$, $36 \in J$. $0.142857 \notin K$.

Just to make matters more confusing than we need them to be, the phrase "is an element of" has an equivalent meaning, namely, "belongs to". Thus we could say that "9 belongs to G, and we would write that as $9 \in G$. Naturally, "is not an element of" can also be expressed as "does not belong to", as in "35 does not belong to A" which we write as $35 \notin A$.

Not only do we use two different expressions, "is an element of" and "belongs to" for the same concept, we actually have a second, slightly different, symbol for $\in$, namely, the symbol ε, and a slightly different symbol for $\notin$, namely $\notin$.